# DE BRUIJN SEQUENCES NOTES

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## 1. The Magic Trick

Here is how the magic trick usually goes: The performer has a deck of cards. He tosses it to one of the spectators and they cut the deck. They toss it to another spectator and they also cut the deck. This goes on until five spectators have cut the cards (or how many is necessary to satisfy the audience). Now each of the five spectators pick the top card and since the deck was cut, the performer doesn't know what the top cards are. Then the performer asks the spectators to visualize their cards and he says he's going to read their minds. The performer starts to struggle and says something along the lines of "There's too much information coming at me so could the people with red cards stand up." Finally, the performer is able to name all the cards. Now the obvious question is: how is this done?

The first step to discovering the secret is to analyze what information the performer is receiving. Well, the only thing he knows is which people have red cards (which also tells him who has black cards). Therefore, the performer has to somehow use it as a code to figure out what the cards are.

**Question 1.1.** *How many different codes are there? In other words, how many different combinations of standing up and sitting down can the performer see?*

**Answer**: There are five spectators and each of them can stand up or sit down. Therefore, there are $2 \times 2 \times 2 \times 2 \times 2 = 32$ possibilities.

Now the performer has to assign these possibilities any set of five cards from the deck. First, it is important to note that the performer uses a 32-card deck.

**Question 1.2.** *How many ways can five people each pick a card from a 32-card deck?*

**Answer**: The first person has 32 choices. Then the second person has 31 choices. This goes on until the fifth person has 28 choices. Therefore, the total number of ways is $32 \times 31 \times 30 \times 29 \times 28 = 24165120$.

This is a lot! There is no way the performer can map the 32 possibilities to 24 million possibilities. Luckily a very nice thing happens in the trick. The performer asks to cut the deck so the order of the deck stays the same. Additionally, the spectators pick the top five cards so five cards in a row. Therefore the question becomes:

**Question 1.3.** *How many ways are there to pick five cards in a row of an ordered 32-card deck?*

**Answer**: The first card can be any of the 32 cards. The next 4 cards are already determined by the first card so the total number of ways is 32.

This is perfect. Now we can 32 ways the spectators can stand up and the 32 ways they can pick five cards.

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