Work and Energy Problems Key

Fun With Fiziks

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Practice Problems Key

1. From conservation of energy, we can write

   $$E_i = E_f \Rightarrow PE_i = KE_f$$

   Therefore, we can solve for the final velocity v_f.

   $$mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 10} = \boxed{14\,m/s}$$

   Notice how this is the same result we get from kinematics! Because acceleration is constant in this problem, we could have used $v_f^2 = v_0^2 + 2a\Delta x$ to get the same answer.

2. We know that Luke is doing work against gravity. Therefore, he is doing positive work to lift Andrew. The work $W = \Delta U$. There is no negative sign because this is not the work done by gravity. Therefore, the work for 1 rep is

   $$W = \Delta U = U_f - U_i = mgh - 0 = 55 \cdot 9.8 \cdot 2 = 1078\,J$$

   Therefore, the amount of work Luke does in 8 reps is $8W = \boxed{8624\,J.}$

3. This is a trick question! Balls A, B, and C all have the same velocity just before they hit the ground. This is because they all start off with the same total energy, so they also must end with the same kinetic energy. The total energy of each ball is

   $$E = mgh + \frac{1}{2}mv^2$$

   Notice how only the magnitude of the initial velocity matters, not the direction. This is because energy is a scalar.

4. The rocket engine does work on the rocket, so we can use the work-energy theorem.

   $$W = \Delta K = K_f - K_i$$

The work the rocket engine does is $W = F_{thrust} \cdot d = 100 \cdot 10 = 1000\ J$. Therefore, the final velocity of the rocket is

$$\frac{1}{2} \cdot 1000 \cdot 10^2 + 1000 = \frac{1}{2} \cdot 1000 \cdot v_f^2 \Rightarrow v_f = \boxed{10.1\ m/s}$$

5. In this problem, we will need to define where the $U = 0$ point is. It doesn't matter where, as long as we stay consistent. Notice that if we set the top of the spring as the zero point, then when the spring is fully compressed, we will have negative potential energy! This is perfectly fine, and it is because the potato is at a "negative" height relative to the top of the spring.

   Note: If that feels strange, we can also set $U = 0$ at the point where the spring is fully compressed. This avoids the negative potential energy, since it will be zero. However, both ways will give the same answer. In this solution, we will set $U = 0$ at the top of the spring.

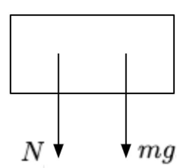
   We can write the conservation of energy equation to solve.

$$U_i + K_i = U_{spring} + U_f \Rightarrow mgh + \frac{1}{2}mv^2 = \frac{1}{2}kx^2 - mgx$$

   The $-mgx$ term comes from when the potato is a height x *below* the top of the spring.

$$1 \cdot 9.8 \cdot 1 + \frac{1}{2} \cdot 1 \cdot 2^2 = \frac{1}{2}k \cdot 0.5^2 - 1 \cdot 9.8 \cdot 0.5 \Rightarrow k = \boxed{133.6\ N/m}$$

6. This problem is a little tricky because it combines forces and energy! We need to look at the forces on the roller coaster cart at the top of the loop to figure out the velocity.



   The roller coaster barely makes it around the loop when $N = 0$. Also, since the centripetal force is the net force, we can write

$$F_{net} = F_c = N + mg = mg \Rightarrow \frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$$

   Now that we have velocity, we can plug it into the conservation of energy equation.

$$U_i = U_f + K_f \Rightarrow mgh = 2mgR + \frac{1}{2}mv^2 = 2mgR + \frac{1}{2}m(\sqrt{gR})^2$$

Solving this for h,

$$h = 2R + \frac{1}{2}R = \frac{5}{2}R = \frac{5}{2} \cdot 10 = \boxed{25\ m}$$

7. In this problem, we are dealing with energy loss through work done by friction W_f.

   (a) We can write the conservation of energy equation, accounting for the loss of energy through friction.

   $$E_i - W_f = E_f \Rightarrow E_i - F_f d = E_i - \mu mgd = E_f$$

   The only type of energy in this problem is kinetic energy. Since Sid comes to a complete stop, $E_f = 0$. Therefore, we can solve for d in the equation.

   $$\frac{1}{2}mv^2 - \mu mgd = 0 \Rightarrow d = \frac{v^2}{2\mu g} = \frac{5^2}{2 \cdot 0.5 \cdot 9.8} = \boxed{2.55\ m}$$

   (b) The force that the rocket booster needs to deliver is equal and opposite to the friction force. So, $F_{thrust} = F_f = \mu mg$. Power is $P = F \cdot v$, so we can plug the expression in and solve.

   $$P = F_{thrust} \cdot v = \mu mgv = 0.5 \cdot 10 \cdot 9.8 \cdot 5 = \boxed{245\ W}$$