Using Physics for Mathematics

Nandana Madhukara

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Agenda

The General Idea

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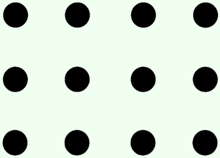
Physical Representation

- Ex: Commutative Property



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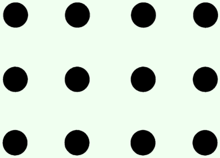
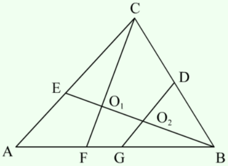


Using Physics

- Ex: Mass Points



Center of Mass Arguments

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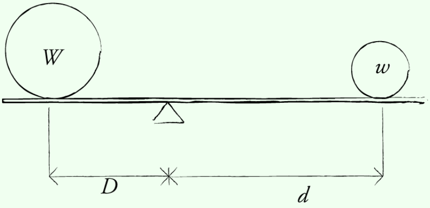
Axioms

1. Law of the lever

$$WD = wd$$



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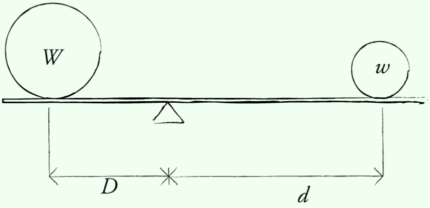

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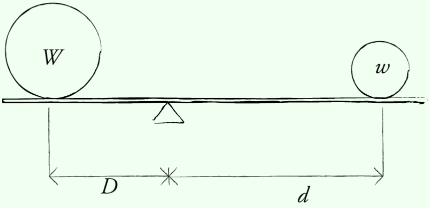

2. Every geometrical object has a CM
3. Mass of CM is sum of masses of its parts.

Medians

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The centroid of a triangle splits a median into two segments with the ratio of its lengths being 2:1

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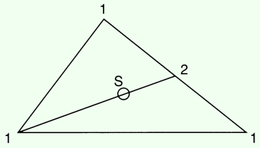

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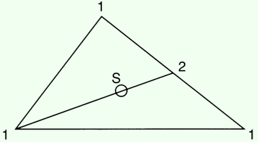

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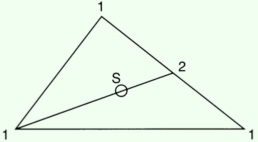

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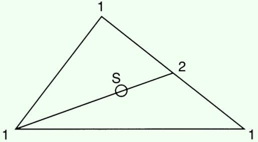

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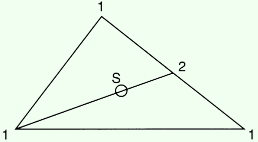

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- Centroid splits median into ratio 2:1

□

Problems

Question

In triangle ABC, E is on AC so that $CE = 3AE$ and F is on AB so that $BF = 3AF$. If BE and CF intersect at O and line AO intersects BC at D, compute $\frac{OB}{OE}$ and $\frac{OD}{OA}$.

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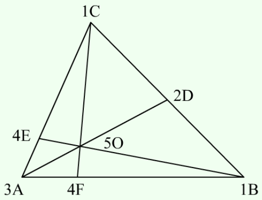
In triangle ABC, E is on AC so that $CE = 3AE$ and F is on AB so that $BF = 3AF$. If BE and CF intersect at O and line AO intersects BC at D, compute $\frac{OB}{OE}$ and $\frac{OD}{OA}$.

Proof.

We use the following masses:



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$$PQ^2 + QR^2 = PR^2.$$

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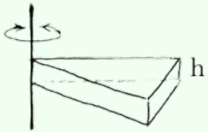
- Consider the following setup with air inside



Pythagorean Theorem (cont.)

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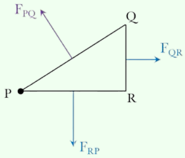
Proof (Cont.)

- The pressure will cause the following forces (aerial view)



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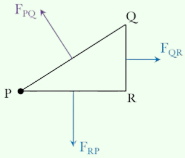

- Box doesn't move so torques must cancel out:

$$F_{PQ} \times PQ/2 = F_{RP} \times PR/2 + F_{QR} \times QR/2$$

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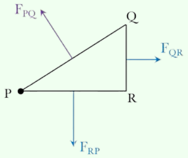

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- If pressure is p,

$$ph(PQ^2)/2 = ph(PR^2)/2 + ph(QR^2/2)$$

□

Pick's Theorem

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Theorem

Let P be a lattice polygon where all its vertices are lattice points. The area of P is the sum of its interior points plus half its boundary points minus 1:

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- Because of symmetry of lattice plane, net flow of water across edge is 0 $\implies$ WLOG assume boundary of P is impermeable
- Each interior point contributes volume 1 and each edge point contribute volume 1/2

□

Pick's Theorem proof contd.

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- Edge points and vertex points make up boundary points
- Therefore,

$$\text{area of } P = |\text{int } P| + 1/2 \cdot |\partial P| - 1$$

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Multiplicative Scoring

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Game Rules

1. Start with a number n
2. Split n into i and $n - i$
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4. Repeat step 2 until you have all 1s

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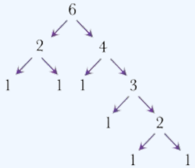

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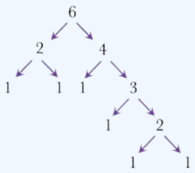

Score:

$$2 \times 4 + 1 \times 1 + 1 \times 3 + 1 \times 2 + 1 \times 1 = 15$$

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- When we split, the δPE_i is our score for the split
- Total score:

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- However

$$\Delta PE = (n - 1) + ... + 2 + 1 = \boxed{\frac{n(n-1)}{2}}$$

□