SIMPLE HARMONIC MOTION HANDOUT

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1. Formulas

Angular frequency of spring:

$$\omega = \sqrt{\frac{k}{m}}.$$

Angular frequency of simple pendulum (with small angle approximation):

$$\omega = \sqrt{\frac{g}{l}}.$$

Angular frequency of pendulum with moment of inertia I about pivot:

$$\omega = \sqrt{\frac{mg\ell_{\text{CM}}}{I}}$$

where ℓ_{CM} is the distance from the pivot to the center of mass. The period is given by $T = 2\pi/\omega$. Energy of spring:

$$E = \frac{1}{2}kx^2.$$

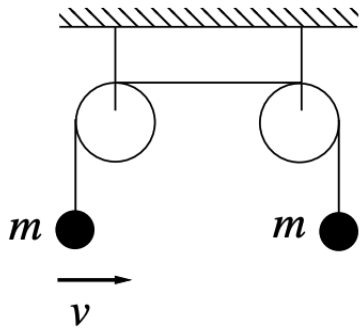
2. Problems

2.1. **Easy.**

(1) A spring is hanging from a ceiling in equilibrium. If the spring is stretched and released, what is the period of oscillation? What is its maximum speed during the oscillation?

(2) A pendulum of length L oscillates inside a box. A person picks up the box and gently shakes it vertically with frequency ω and a fixed amplitude for a fixed time. To maximize the final amplitude of the pendulum, ω should satisfy what?

(3) Two particles of mass m are connected by pulleys as shown.



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The mass on the left is given a small horizontal velocity, and oscillates back and forth. The mass on the right

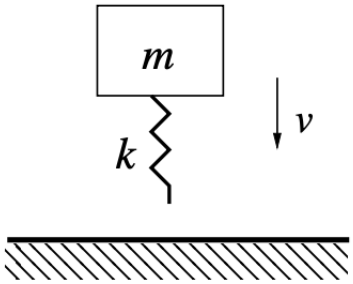
(A) remains at rest
(B) oscillates vertically, and with a net upward motion
(C) oscillates vertically, and with a net downward motion
(D) oscillates vertically, with no net motion
(E) oscillates horizontally, with no net motion

(4) A light, uniform, ideal spring is fixed at one end. If a mass is attached to the other end, the system oscillates with angular frequency ω. Now suppose the spring is fixed at the other end, then cut in half. The mass is attached between the two half springs.

What is the new angular frequency of oscillations?

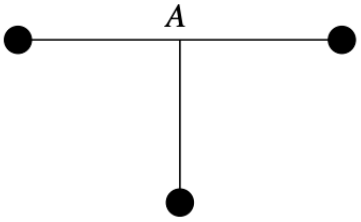
2.2. **Hard.**

(1) A pogo stick is modeled as a massless spring of spring constant k attached to the bottom of a block of mass m. The pogo stick is dropped with the spring pointing downward and hits the ground with speed v. At the moment of the collision, the free end of the spring sticks permanently to the ground.



During the subsequent oscillations, what is the maximum speed of the block

(2) Three identical masses are connected with identical rigid rods and pivoted at point A. If the lowest mass receives a small horizontal push to the left, it oscillates with period T_1. If it instead receives a small push into the page, it oscillates with period T_2. What is the ratio T_1/T_2?



(3) A uniform bar of length L and mass M is supported by a fixed pivot a distance x from its center. The bar is released from rest from a horizontal position. For what x is the period of osscilation minimal?