Momentum

Fun With Fiziks

What is momentum?

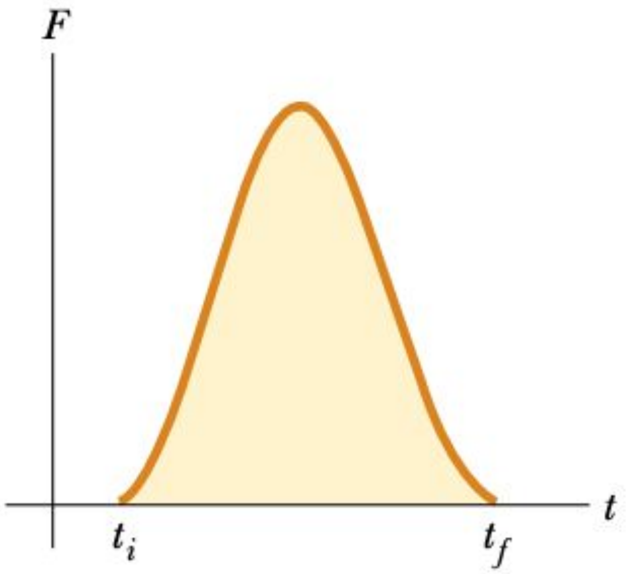
- Basically how hard something is to stop
- Represent momentum using ρ
- Is a vector quantity (direction/magnitude)
- Units: kg*m/s
- $\rho = mv$

Conservation of momentum

- Just like energy, momentum is conserved!
- Initial momentum = final momentum
    - Momentum is only conserved when there are no external forces! (just like energy)
- Can write conservation of momentum as:
    - $\rho_i = \rho_f$ = constant
    - $\rho_{1i} + \rho_{2i} = \rho_{1f} + \rho_{2f}$

What is impulse?

- Represent impulse using I
- I = FΔt
  - Analogous to work
- Area under force time curv
- Is a vector
- Units: kg*m/s



Impulse cont.

- Impulse momentum theorem: $I = \Delta p$
- Derivation
  - $I = F\Delta t = ma\Delta t = m\Delta v = \Delta p$
- This looks similar to work-energy theorem!!
  - Momentum - related to time
  - Work - related to distance

Energy vs Momentum

Energy

- Measured in Joules
- $KE=1/2mv^2$
- $PE = mgh$
- Scalar, meaning quantity of energy will be always positive regardless of direction
- Conserved in elastic collision
- Not conserved in inelastic collision

Momentum

- Measured in kg m/s
- $p=mv$
- Vector, meaning quantity can be either positive or negative depending on the direction of the object's motion
- Conserved in all situation

Practice problem

Bob throws a 10 kg ball at Joe at 100 m/s and Joe hits it back at Bob at 1 m/s. What is the magnitude of the impulse on the ball from Joe?

Use $I = \Delta\rho$

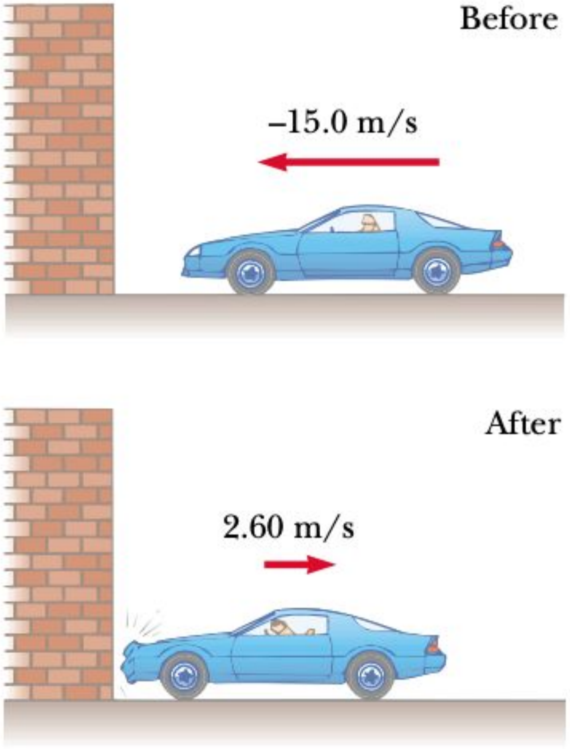
$I = \Delta\rho = \rho_f - \rho_i = m_{ball}v_f - m_{ball}v_i$

Plug in numbers:

$I = \Delta\rho = 10*1 - 10*(-100) = 1010\ kg*m/s$

Another practice problem

In a crash test, a car of mass 1500 kg collides with a wall. The initial and final velocities are 15 m/s and 2.6 m/s, respectively. If the collision lasts for 0.15 s, find the impulse caused by the collision and the force exerted on the car.



- Use both definitions of impulse
  - $I = \Delta\rho$
  - $I = F_{net}\Delta t$
- $I = \Delta\rho = mv_f - mv_i = 1500*2.6 - 1500*(-15)$
  - I = 2.64*10^4 kg*m/s
- $I = F_{net}\Delta t = 2.64*10^4 = F_{net}*0.15$
  - F_{net} = 1.76*10^5 N

When do we use momentum (the most)?

COLLISIONS!

Types of collisions

- Elastic: KE and momentum conserved (impossible)
- Inelastic: KE not constant because energy lost
- Perfectly inelastic: objects stick together after colliding (car crash)
- Momentum is constant in all collisions, but KE is not

Elastic collisions

- Momentum conserved:
- $\rho_{1i} + \rho_{2i} = \rho_{1f} + \rho_{2f}$
- Kinetic energy is also conserved:
- $1/2m_1v_{1i}$^2 + $1/2m_2v_{2i}$^2 = $1/2m_1v_{1f}$^2 + $1/2m_2v_{2f}$^2

Practice problem

A 2 kg ball is moving right at 2 m/s and a 3 kg ball moving left at 3 m/s collide. The collision is elastic. Find the velocity of the 2 kg ball.

Plug in numbers in $\rho_{1i} + \rho_{2i} = \rho_{1f} + \rho_{2f}$

$\rho_{1i} = 2*2$, $\rho_{2i} = 3*(-3)$, $\rho_{2f} = ?$, $\rho_{1f} = ?$

$-5 = m_1v_{1f} + m_2v_{1f}$

½*2*4 + ½*3*9 = 17.5 = $1/2m_1v_{1f}$^2+ $1/2m_2v_{2f}$^2

$v_{1f} = -4$ m/s, which means the ball goes 4 m/s to the left

What if you don't know either of the final speeds?

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i}$$

Inelastic collisions

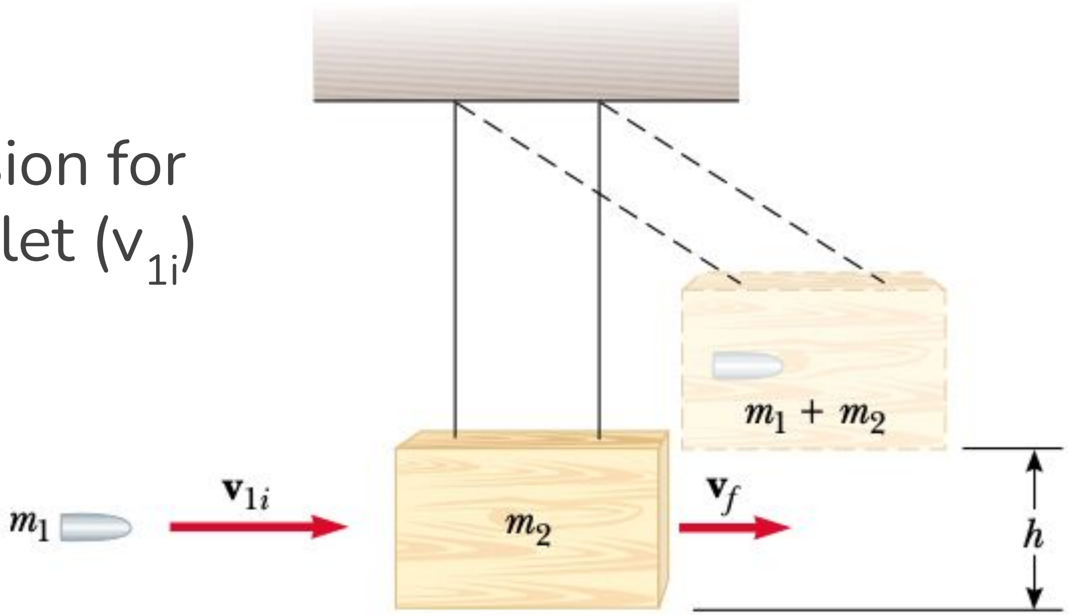
- What happens in reality - energy is not conserved because there is friction
- Momentum is still conserved
- We don’t really deal with inelastic collisions much

Perfectly inelastic collisions

- When two things stick together after a collision
- The conservation of momentum equation:
- $m_1v_{1i} + m_2v_{2i} = (m_1 + m_2)*v_f$

Ballistic pendulum

Determine an expression for the velocity of the bullet (v_{1i})



- Collision is perfectly inelastic
- m_1 = mass of bullet, m_2 = mass of block, m_{1i} = initial velocity of bullet, v_{2i} = initial velocity of block, v_f = final velocity
- $v_{2i} = 0$

- $m_1 v_{1i} = (m_1 + m_2) * v_f$
- $KE_f = 1/2(m_1 + m_2) * v_f ^2$
- Solve momentum equation for $v_f = m_1 v_{1i} / (m_1 + m_2)$
- Plug in v_f into KE equation

$$K_f = \frac{{m_1}^2 {v_1}_i^2}{2(m_1 + m_2)}$$

- All KE becomes PE (conservation of energy)
  ⇒ KE = PE

$$\frac{{m_1}^2{v_{1i}}^2}{2(m_1 + m_2)} = (m_1 + m_2)gh$$

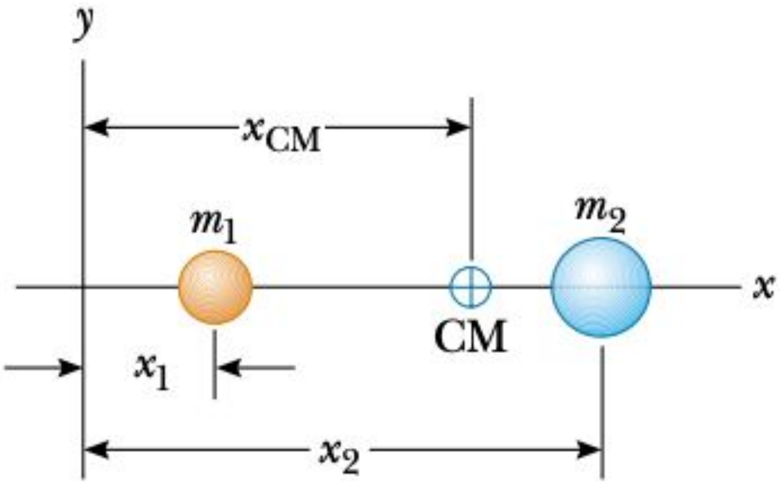
- Solve for v_{1i}

$$v_{1i} = \left(\frac{m_1 + m_2}{m_1}\right)\sqrt{2gh}$$

Center of Mass

What is CM?

- Where something balances on 1 finger
- Average location of all the mass in an object



$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

General formula

$$x_{CM} \equiv \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots + m_n x_n}{m_1 + m_2 + m_3 + \cdots + m_n} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

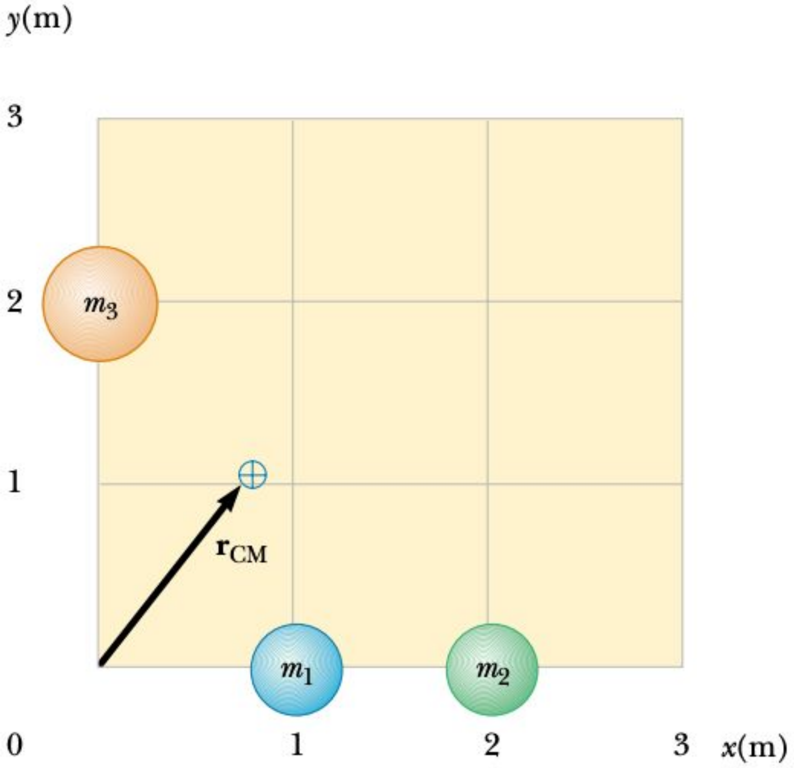
- This is x coordinate of CM
- Do the same but use y or z coordinates to find y and z coordinate of CM
- Basically multiply mass by the position for all points, add them up, and divide by total mass

Practice Problems

Find center of mass of this system of 3 particles:

$m_1 = m_2 = 1$ kg

$m_3 = 2$ kg



Answer

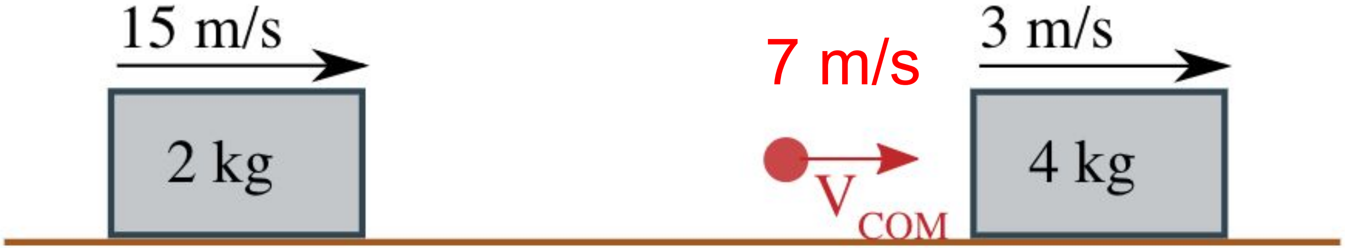
- Total mass = 1 kg + 1 kg + 2 kg = 4 kg
- x_{cm} = 1*1 + 1*2 + 2*0/4 = 0.75 m
- y_{cm} = 1*0 + 1*0 + 2*2/4 = 1 m
- Coordinates of center of mass: (x_{cm}, y_{cm}) = (0.75 m, 1 m)
- Vector of center of mass: <0.75 m, 1 m>

Center of gravity

- Very closely related to CM
- CG is the point at which gravity acts on an object
- CG = CM in a uniform gravitational field, which is assumed in mechanics problems

How is center of mass related to momentum?

- When multiple objects move at constant velocities, their center of mass is moving at v_{cm}
- v_{cm} = sum of momentums/total mass
- Makes solving momentum problems easier

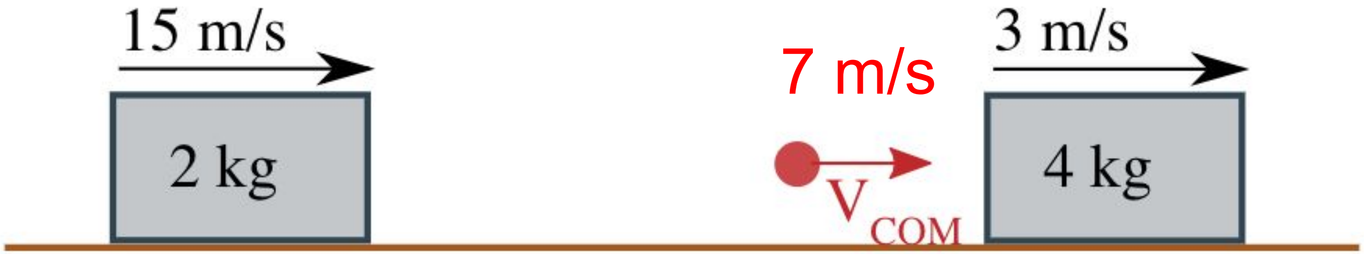


Make v_{cm} = 0 to solve problems

- To make $v_{cm} = 0$, we have to “scale” the other velocities
- Subtract v_{cm} (7 m/s in this case) from all the velocities
- When $v_{cm} = 0$, velocities are the opposites after collision
- To get the actual final velocities, add back v_{cm}

-8 m/s + 7 m/s = -1 m/s

4 m/s + 7 m/s = 11 m/s



Check the previous problem using the formula

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_{2i}$$

This is really hard to memorize

$$v_{1f} = \left(\frac{2-4}{2+4}\right)\cdot\ 15 + \left(\frac{2\cdot\ 4}{2+4}\right)\cdot\ 3 = -1$$

$$v_{2f} = \left(\frac{2\cdot\ 2}{6}\right)\cdot\ 15 + \left(\frac{2}{6}\right)\cdot\ 3 = 11$$

But we get the same answer as the $v_{cm} = 0$ method!

2D Collisions

2D collisions

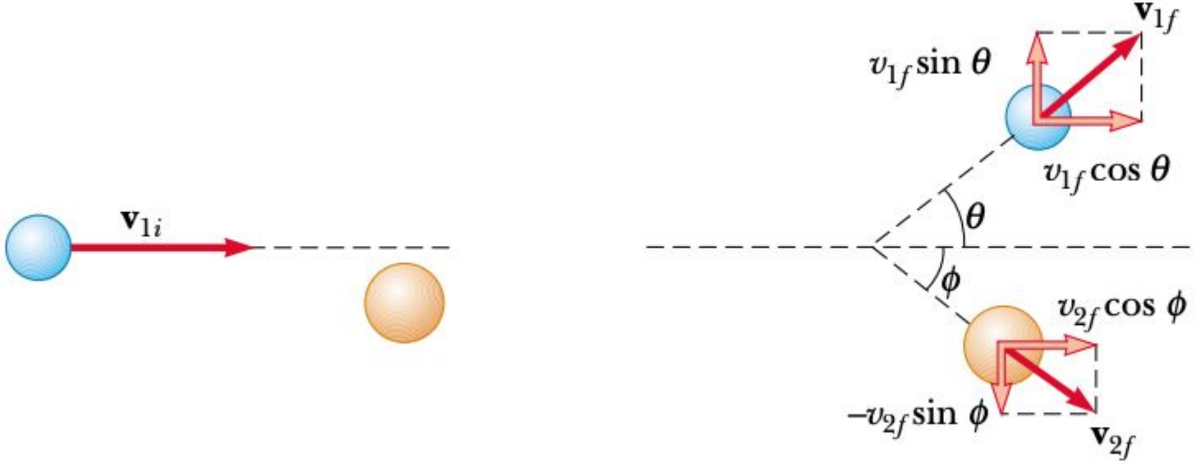
- Same as 1D collisions but solve for x and y components separately
- Since velocity is vector, split vector into x and y components

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Glancing collisions

- One particle is moving and another is at rest
- After, the two particles move at an angle



(a) Before the collision (b) After the collision

Glancing collisions cont.

- Since $v_{2i} = 0$, m_2v_{2i} term = 0
- x components of final velocity are mvcosθ
- Since initially the particles aren't moving up or down, momentum in y direction initial and final is 0

$$m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

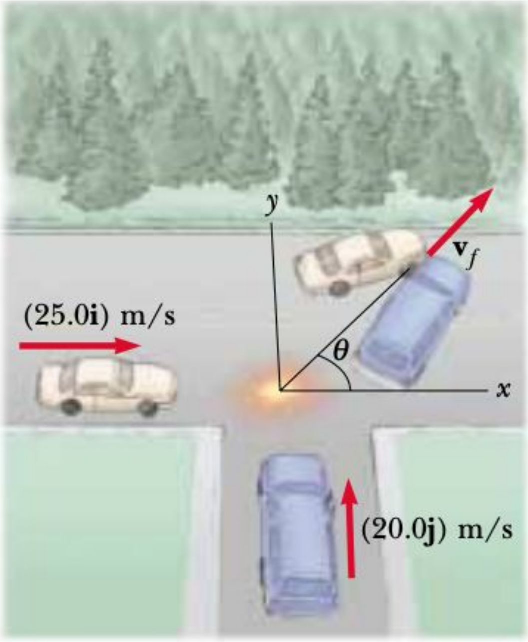
$$0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$

Practice problem

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg van traveling north at a speed of 20.0 m/s, as shown in Figure 9.15. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision (that is, they stick together).

Direction: $\theta = 53.1^{\circ}$

Magnitude: $v_f = 15.6$ m/s



Explanation

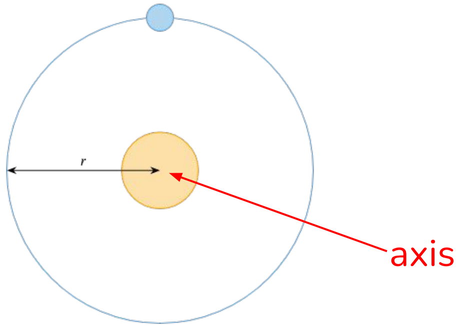
- Do x and y components of velocity separately
- Momentum in x direction:
  - $1500*25 = (1500 + 2500)*v_f*\cos\theta$
- Momentum in y direction:
  - $2500*20 = (1500 + 2500)*v_f*\sin\theta$
- Have 2 unknowns and 2 equations so solve for θ and v_f

- Divide 2 equations to cancel out v_f solve for θ
  - $2500*20/1500*25 = (1500 + 2500)*v_f*\sin\theta/(1500 + 2500)*v_f*\cos\theta$
  - $1.33 = \tan\theta$
  - $\theta = 53.1^\circ$
- Plug in 53.1° for θ in either equation to solve for v_f
  - $1500*25 = (1500 + 2500)*v_f*\cos(53.1)$
  - $v_f = 15.6$ m/s

Rotational Motion

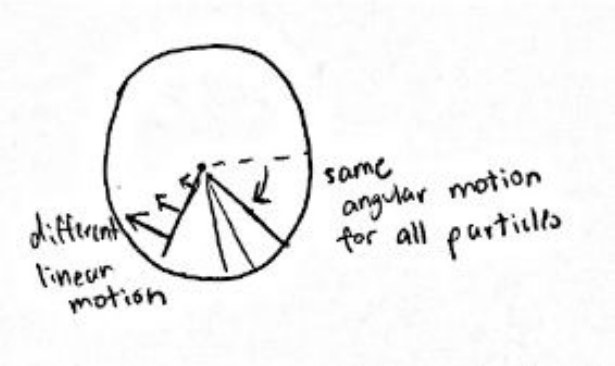
What is rotation?

- Rotation is when an object moves around a stationary point.
- This stationary point is called the axis.
- All points on the object move around the axis in a circular path.



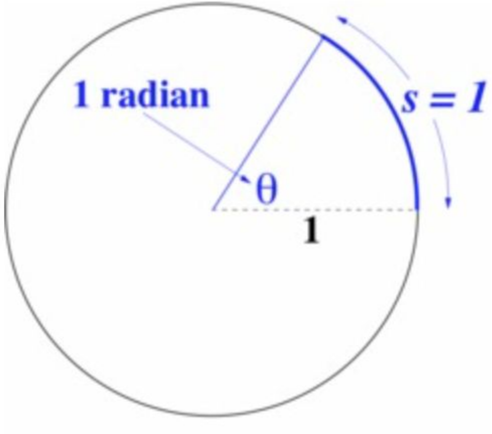
“Angular”

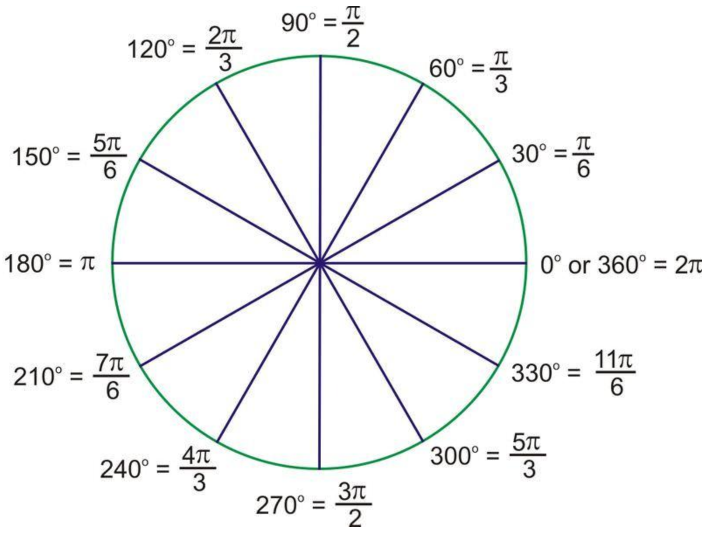
- When describing rotations, we use the word “angular” (angular momentum, angular velocity, angular acceleration, etc.).
- Unlike linear motion, during a rotation different parts of the object move at different velocities. However, every particle rotates through the same **angle,** hence the word “angular”.



Radians

- Another way to measure angles
- Usually preferred compared to degrees
- If you walk 1 unit along a unit circle, the angle you have traveled is 1 radian!
- Basically however far you walk along the circle is your angle in radians





Why do we use radians?

- It makes it easy to convert between linear and angular quantities
- We know on a unit circle, s = θ
  - How far you walk = angle
- For a circle of radius r, s = rθ
  - You walk r times further but the angle you travel still stays the same

- Displacements $s = \theta r$
- Speeds $v = \omega r$
- Accelerations $a = \alpha r$

Angular vs linear quantities

- Basically the same as normal kinematics, just replace the linear quantity with the corresponding angular quantity

Linear quantity	symb.	Angular quantity	symb.
distance	d	angle	θ
velocity	v	angular vel.	ω
acceleration	a	angular accel.	α

Kinematics for Rotations

Translational Motion	Rotational Motion
$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
$\Delta x = \frac{1}{2}at^2 + v_i t$	$\Delta\theta = \frac{1}{2}\alpha t^2 + \omega_i t$
$\Delta x = \frac{1}{2}(v_f + v_i)t$	$\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)t$
$v_f^2 = v_i^2 + 2a\Delta x$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$

Angular momentum

- The rotational version of normal momentum
- If momentum = how hard something is to stop moving, then rotational momentum = how hard something is to stop rotating

Angular Momentum = Moment of Inertia x Angular Velocity

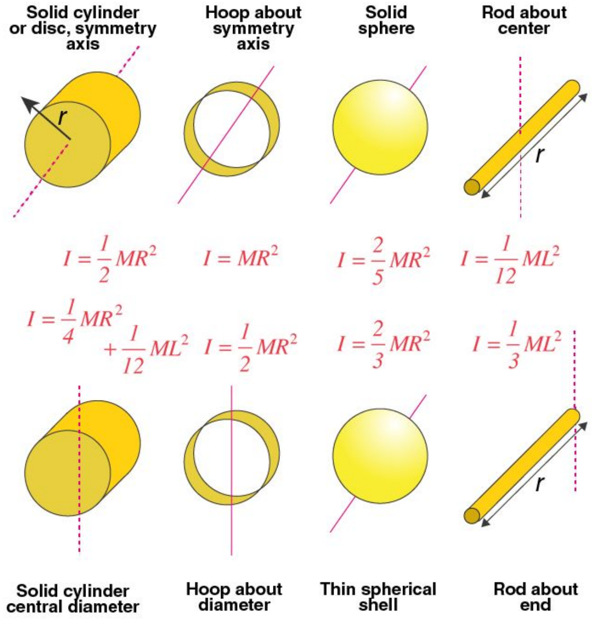
$$L = I \times \omega$$

Linear Momentum = Mass x Velocity

$$p = m \times v$$

What is the moment of inertia?

- Rotational version of mass
- How hard something is to spin
  - Normal mass: more mass → harder to move
  - Moment of inertia: more rotational inertia → harder to rotate
- Different from normal mass because it depends on the **shape of the object** and the **axis of rotation**



Moment of inertia cont.

- The moment of inertia depends on the radius
  - If R increases, then rotational inertia increases
- This tells us that the further the mass is from the axis of rotation, the harder something is to spin