Momentum Team Problems Key

Fun With Fiziks

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Practice Problems Key

1. This is a perfectly inelastic collision. Therefore, we can use conservation of momentum to solve for their final velocity. Even though we don't know their masses, it will cancel out.

   $$p_E + p_J = p_f \Rightarrow 10m - 2m = 2mv_f \Rightarrow v_f = \boxed{4\,m/s}$$

   Notice that Jonathan's momentum is $p_J = -2m$ since he is moving in the opposite direction as Erica!

2. We can use the impulse-momentum theorem since we are given $t = 2\,s$ and want to figure out F.

   $$I = Ft = \Delta p = 0 - mv_i \Rightarrow F = -\frac{mv_i}{t} = -3\,N$$

   This force is the force of Chris's leg on the stick. Therefore, force of the stick on Chris's leg is equal and opposite by Newton's Third Law, so $F = \boxed{3\,N}$.

3. This is a 2D momentum problem, so we need to split the momentum into x and y components. In the x direction,

   $$m_J v_J = (m_J + m_A)v_f \cos\theta$$

   and in the y direction,

   $$m_A v_A = (m_J + m_A)v_f \sin\theta$$

   Plugging in the numbers, we get the following system of equations:

   $$360 = (60 + m_A) \cdot 3\cos 45^\circ$$

   $$m_A v_A = (60 + m_A) \cdot 3\cos 45^\circ$$

   From the first equation, we see that $m_A = 109.7\,kg$. Plugging this into the second equation and solving, we get that $v_A = 3.28\,m/s$. Therefore, $\boxed{\text{Arone needs to weigh } 109.7\,kg \text{ and needs to run at } 3.28\,m/s.}$

4. We can use the impulse-momentum theorem since we are given $t = 10\, s$ and $F = 100\, N$.

$$I = Ft = \Delta p = mv_f - mv_i \Rightarrow v_f = \frac{Ft + mv_i}{m} = \boxed{11\, m/s}$$

Note: This problem is similar to the rocket question in the work and energy problem set, except we are given time instead of distance. This problem shows that when there are **forces over time**, we want to use $Ft = \Delta p$ (**impulse-momentum**), and if there are **forces over distances**, we want to use $Fd = \Delta K$ (**work-energy**).

5. Since this is a collision problem, can use conservation of momentum to solve for v_f. So, we have $p_{A,i} + p_{B,i} = p_{A,f} + p_{B,f}$. Plugging in numbers,

$$3 \cdot 1 - 1 \cdot 2 = -3 \cdot 0.5 + 1 \cdot v_f \Rightarrow v_f = \boxed{2.5\, m/s}$$

Notice that $p_{B,i}$ and $p_{A,f}$ are both negative, since the blocks are moving to the left.

Note: As an extension to this problem, we can compare the kinetic energies before and after the collision:

$$K_i = \frac{1}{2} \cdot 3 \cdot 1^2 + \frac{1}{2} \cdot 1 \cdot 2^2 = 3.5$$
$$K_f = \frac{1}{2} \cdot 3 \cdot 0.5^2 + \frac{1}{2} \cdot 1 \cdot 2.5^2 = 3.5$$

Since $K_i = K_f$, energy is conserved, so this collision is elastic! However, this is just a coincidence. When you just solve the conservation of momentum equation, you aren't guaranteed to get numbers that make the collision elastic.

6. This problem is tricky because it isn't really clear how we can apply conservation of momentum to solve it. Notice that when the firework explodes at the top of the trajectory, it is not moving, so $p_i = 0$. Then, after the firework explodes, momentum is conserved because there are no external forces (the firework explodes because of internal forces). Therefore, $p_f = p_i = 0$. This means that the total momentum of the red, white, and blue particles must be 0. Since the particles are moving in 2D, we need to split the momentum into x and y components. In the x direction.

$$0 = 10m + mv_x \Rightarrow v_x = -10\, m/s$$

and in the y direction,

$$0 = 10m + mv_y \Rightarrow v_y = -10\, m/s$$

Therefore, the blue particle moves diagonally down and to the left to cancel out the other two particles. The velocity is $v = \sqrt{v_x^2 + v_y^2} = \boxed{14.1\, m/s}$ with a direction of $\boxed{45^\circ \text{ below the horizontal.}}$