Kinematics

Fun With Fiziks

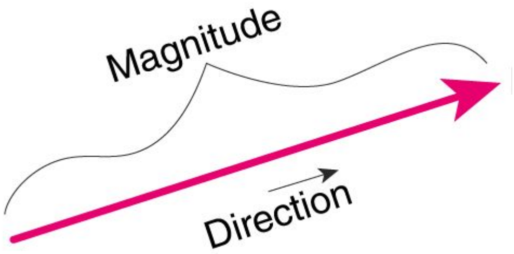
What is kinematics?

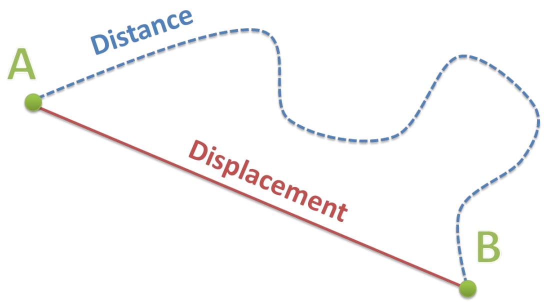
- Basically describes how objects move
- Useful quantities (variable names don't matter):
  - **x**: position (m)
  - **Δx** = x_f - x_i: displacement (m)
  - **v**: velocity (m/s)
  - **a**: acceleration (m/s^2)
  - **t**: time (s)
- There are 4 kinematic equations that relate the quantities above
- Only can be used when acceleration is constant
- **But why do these equations make sense?**

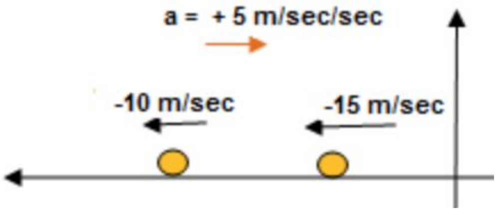
1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

Vector and scalar quantities

- Very important in kinematics and beyond
  - **Scalars**: normal numbers
  - **Vectors**: magnitude and direction (arrow)
- Vector quantities: position, velocity, acceleration
- Scalar quantities: *displacement*, *speed*, time
  - **Displacement is the magnitude of Δx - straight line distance between positions**
  - **Speed is the magnitude of velocity**
- How do we represent direction?
  - The sign of the vector quantity
  - Ex: velocity pointing right is +, left is -



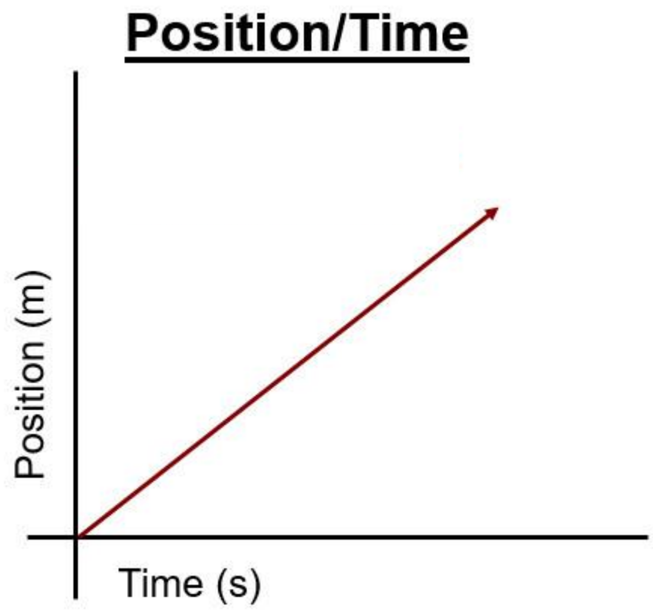




Relating the variables

- How can we relate velocity to the other quantities (Δx, a, t)?
- How can we relate acceleration to the other quantities (Δx, v, t)?

Position v time graphs



- We know that $x = vt$
- Equation of a line ($y = mx + b$):
  - m = velocity
  - What does the y intercept represent?

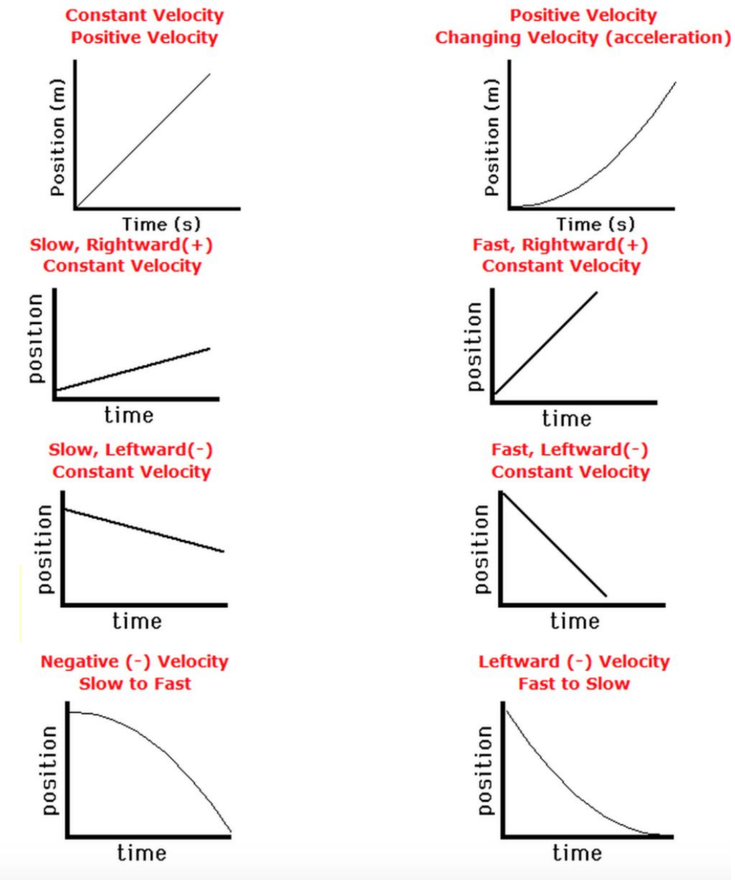
- The position vs time graph is linear when $a = 0 \rightarrow v$ is constant $\rightarrow$ slope is constant
- $1/2at^2$ term goes to 0

Position v time graphs

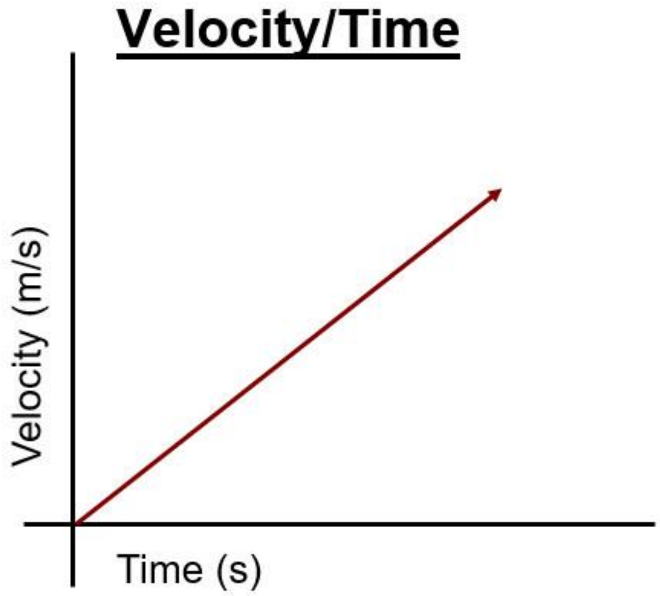
- What if $a \neq 0$?
  - We need to add an extra $1/2at^2$ term to $\Delta x = vt$ to make it work
  - Why $1/2at^2$? Calculus!

$$3. \quad \Delta x = v_0 t + \frac{1}{2} a t^2$$

- Position time graph is a quadratic
- Intuitively the graph should keep getting steeper or shallower
- a ⟶ change in v ⟶ change in slope



Velocity v time graphs



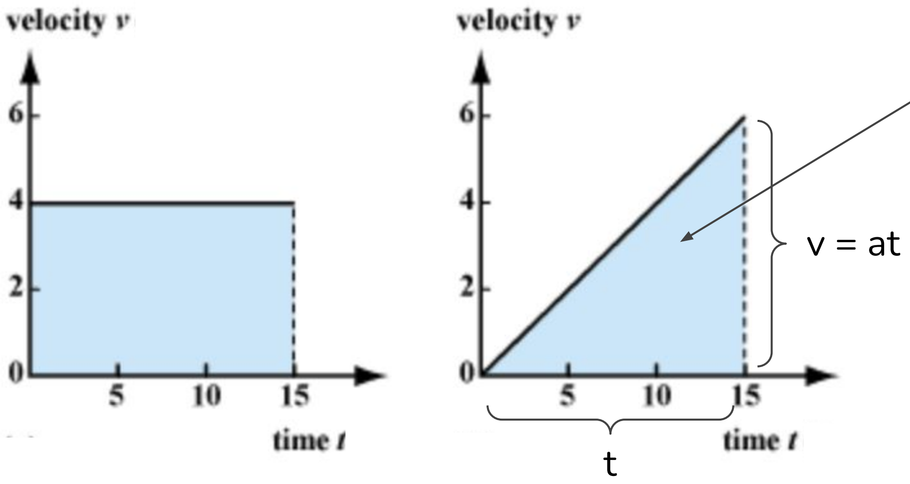
- Similar to position v time graphs
- We know v = at
- Of the form y = mx + b again
  - m = acceleration
  - b = initial velocity

1. $v = v_0 + at$ ✓

- Velocity v time graph is linear when a is constant (a will basically always be constant in kinematics)

Finding distance traveled - why $1/2at^2$?

- We know that to get distance traveled, we multiply velocity by time ($\Delta x = vt$)
- **Distance traveled is the area under the velocity v time graph!**



When there is **constant** acceleration, distance traveled is the area of this triangle

$$\Delta x = A = \frac{1}{2}bh$$

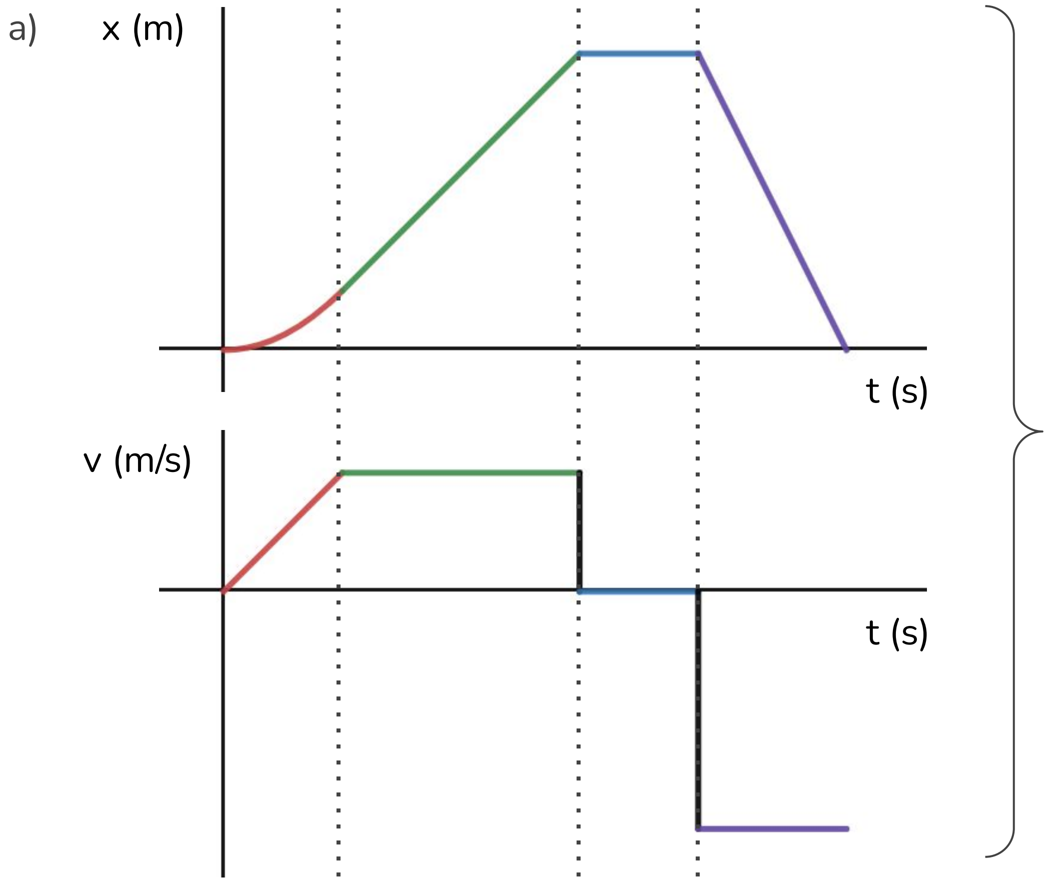
$$= \frac{1}{2}t * at = \boxed{\frac{1}{2}at^2}$$

Practice problem #1

Jonathan loves being everyone's personal uber driver! This time, he is driving from his house to Lulu's house. Jonathan accelerates at 1 m/s^2 from rest for 3 seconds. Then, he continues at this velocity for another 5 minutes. He then stops at Lulu's house for 2 minutes. Then, he drives home at a constant velocity in 6 minutes.

a) Sketch the position vs time and velocity vs time graphs for this scenario.
b) How fast is he going after he accelerates for 3 seconds?
c) How far is Lulu's house from Jonathan's house?

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$



Notice that the v-t graph represents the **slope** of the x-t graph

“Starts from rest” means $v_0 = 0$

b) $v = v_0 + at$

$v = 0\,m/s + 1\,m/s^2 * 3\,s = \boxed{3\,m/s}$

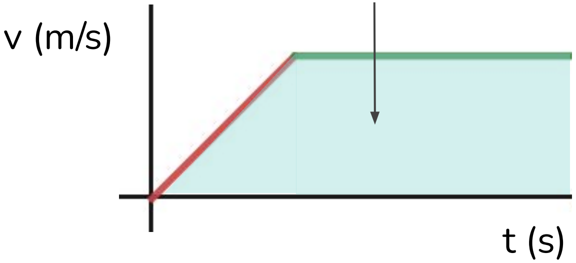
c) $\Delta x = v_0 t + \frac{1}{2}at^2$

$\Delta x_1 = 0\,m/s * 3\,s + \frac{1}{2}(1\,m/s^2)(3\,s)^2 = \frac{9}{2}m = 4.5\,m$

$\Delta x_2 = 3\,m/s * 300\,s + \frac{1}{2}(0\,m/s^2)(300\,s)^2 = 900\,m$

$\Delta x = \Delta x_2 + \Delta x_1 = \boxed{904.5\,m}$

Or, you can find the area of this region



Practice problem #2

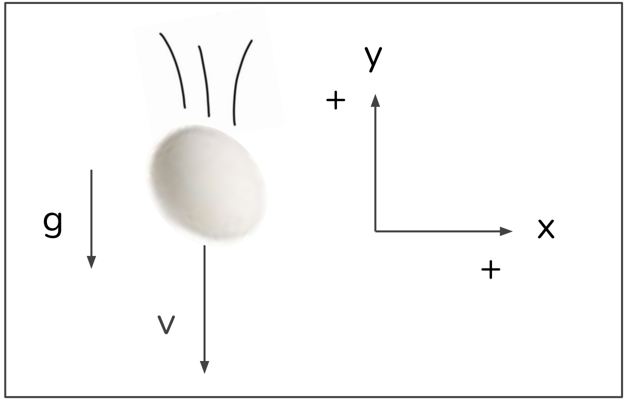
For the school egg drop project, Chris wants to calculate if her design will survive. She guesstimates that if the device is traveling at 2 m/s by the time it hits the ground, the egg will break. What is the max height Chris can drop her device so that the egg survives?

*Note: Earth's gravitational field will accelerate objects at **g = 9.8 m/s^2** (you should remember the constant g, since it comes up very often!)

Hint: this means a = g = 9.8 m/s^2, which applies to all free fall problems

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

Define coordinate axes:



Because v and a are pointing down, they should be negative (they are vectors)

It doesn't really matter how you define which way is positive, **as long as you are consistent!**

Knowns:

- $v_0 = 0\, m/s$
- $a =- g =- 9.8\, m/s^2$
- $v = 2\, m/s$

You can sometimes use logic to figure out what v_0 and v_f are

Unknowns:

- $t = ?\ s$
- $\Delta y = ?\ m$

$$\Delta y = v_0 t + \frac{1}{2}at^2$$

$$v = v_0 + at \Rightarrow t = \frac{v-v_0}{a}$$

$$\Delta y = v_0(\frac{v-v_0}{a}) + \frac{1}{2}a(\frac{v-v_0}{a})^2 = \frac{v_0 v - v_0^2}{a} + \frac{v^2-2v_0v+v_0^2}{2a}$$

$$= \frac{2v_0v-2v_0^2}{2a} + \frac{v^2-2v_0v+v_0^2}{2a} = \frac{2v_0v-2v_0^2+v^2-2v_0v+v_0^2}{2a} = \frac{v^2-v_0^2}{2a}$$

$$\Delta y = \frac{v^2-v_0^2}{2a} \Rightarrow v^2 - v_0^2 = 2a\Delta y \Rightarrow v^2 = v_0^2 + 2a\Delta y$$

4. $v^2 = v_0^2 + 2a\Delta x$

Plugging numbers in:

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{v^2 - v_0^2}{-2g} = -\frac{(2\,m/s)^2 - (0\,m/s)^2}{2 * 9.8\,m/s^2} = -\ 0.2\,m$$

$$|\Delta y| = \boxed{0.2\,m}$$

Where did the negative sign go?

- Position is also a vector
- Δy = -0.2 m means that the egg moves 0.2 m **down**
- But we usually give the positive version of the answer |Δy|
- Represents the **magnitude** of the change in position → displacement

Because the egg drop can fall 0.2 m, Chris needs to drop it at a height of 0.2 m.

Deriving the last kinematic equation

- Let's start with Eq 3 (because it looks similar)

$$3. \quad \Delta x = v_0 t + \frac{1}{2} a t^2$$

- We want to get rid of acceleration because its not in Eq 2

$$\Delta x = v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + at \Rightarrow a = \frac{v - v_0}{t}$$

$$\Delta x = v_0 t + \frac{1}{2} \left(\frac{v - v_0}{t}\right) t^2 = v_0 t + \left(\frac{v - v_0}{2}\right) t$$

$$= \frac{2 v_0 t}{2} + \frac{(v - v_0) t}{2} = \left(\frac{2 v_0 + v - v_0}{2}\right) t = \boxed{\left(\frac{v + v_0}{2}\right) t}$$

Goal:

$$2. \quad \Delta x = \left(\frac{v + v_0}{2}\right) t$$

✓

Which equation do I use?

- It's useful to list out the things you know
- Also list out the things you don't know
- Match the known and unknown information with the appropriate equation

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

Given v_0, v_f, a, find Δx

Given t, Δx, v_f, find v_0

Given v_0, Δx, a, find t

Take away: you will always be given 3 pieces of info and asked to solve for a 4th variable → match the equation to the knowns

Which equation do I use?

Equation	x	a	v	v_0	t
$v_f = v_0 + at$	-	√	√	√	√
$\Delta x = \left(\frac{v_0 + v_f}{2}\right) \cdot t$	√	-	√	√	√
$\Delta x = v_0 t + \frac{1}{2} a t^2$	√	√	-	√	√
$v_f^2 = v_0^2 + 2 \cdot a \cdot \Delta x$	√	√	√	√	-

Practice problem #3

Erica was chilling one day when she suddenly hears Justin playing MF DOOM on Spotify 5 m away. Unable to stand the music, she decides to run up behind Justin and slap him. If Erica starts from rest and wants to get to Justin in 1 second to stop the music as quickly as possible, what is her acceleration?

Knowns:

- $\Delta x = 5\,m$
- $v_0 = 0\,m/s$
- $t = 1\,s$

Unknowns:

- $a = ?\,m/s^2$

$$\Delta x = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2$$

$$a = \frac{2\Delta x}{t^2} = \frac{2*5\,m}{(1\,s)^2} = \boxed{10\,m/s^2}$$

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\boxed{\Delta x = v_0 t + \frac{1}{2}at^2}$
4. $v^2 = v_0^2 + 2a\Delta x$

Practice problem #4

Luke wants to top Chris's egg drop device. Instead of failing to save the egg, his strategy is to successfully break the egg. Luke tests his device off a 10 m high balcony. Luke guesstimates that his egg will break if it is traveling at 15 m/s by the time it hits the ground. To achieve this speed, Luke needs to throw the device off the balcony at some initial velocity. What is the initial velocity required?

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

Knowns:

- $\Delta y = -10\,m$
- $v_f = -15\,m/s$
- $a = -g = -9.8\,m/s^2$

Unknowns:

- $v_0 = \ ?\ m/s$

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

$$v_f^2 = v_0^2 + 2a\Delta y \Rightarrow v_0 = \sqrt{v_f^2 - 2a\Delta y}$$

$$v_0 = \sqrt{v_f^2 - 2a\Delta y}$$

$$= \sqrt{(-15\,m/s)^2 - 2(-9.8\,m/s^2)(-10\,m)}$$

$$= -5.39\,m/s$$

$$|v_0| = 5.39\,m/s$$

- Notice how all the negative signs cancel out
  - This means you could've defined **down as positive**
- It turns out that the direction of v_0 doesn't actually matter, only the magnitude (we will see why when we learn about energy)!

Extension

Let's say that Luke's egg breaks if the device travels at 10 m/s by the time it hits the ground. Is it possible for Luke to throw the device in any direction (up or down) at any initial velocity and have the egg survive?

$$v_0 = \sqrt{v_f^2 - 2a\Delta y}$$

$$= \sqrt{(10\, m/s)^2 - 2(-9.8\, m/s^2)(-10\, m)}$$

$$= \boxed{\sqrt{-96}}$$

- It is not possible! When we try and plug in numbers, we get a negative in the square root :(

What does this mean physically?

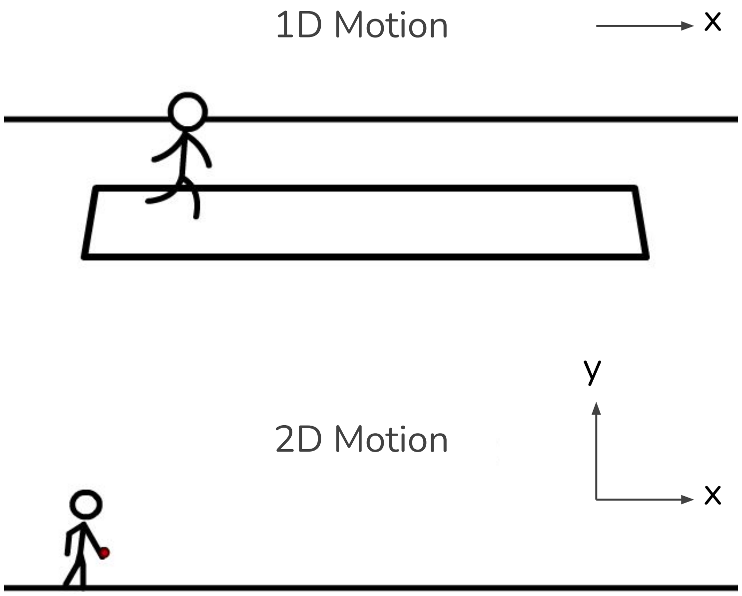
- If your egg drop device is too weak, it will break no matter how you drop it

Kinematics in 2D

- Describes motion in 2D - x and y directions
- Luckily, we can **split motion into x and y components and deal with them separately**

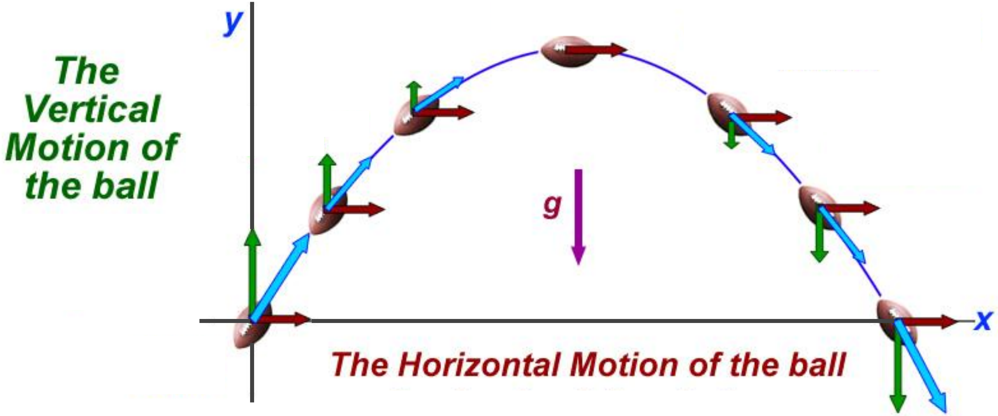
- All of the kinematic equations we learned before still apply :D

1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$



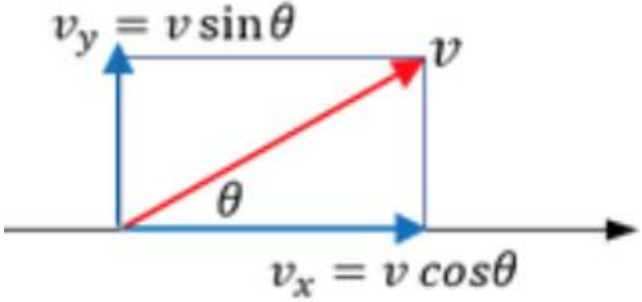
The big picture idea

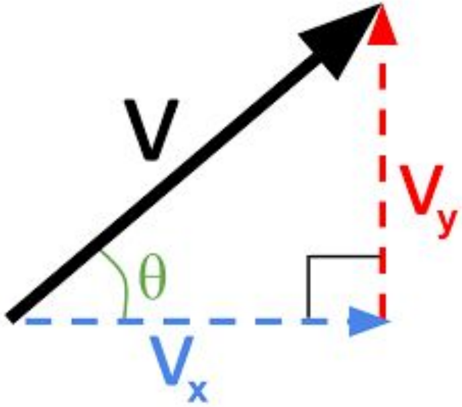
- Let's look at a special type of 2D kinematics - **projectile motion**
- A few important things:
  - The x velocity is constant! (why?)
  - The y velocity is not constant because there is gravitational acceleration (g = 9.8)
  - **The x and y motion do not affect each other**



Applying trig to kinematics

- Use sin and cos to figure out the x and y components of velocity
- We want to do this so we can look at x and y directions separately

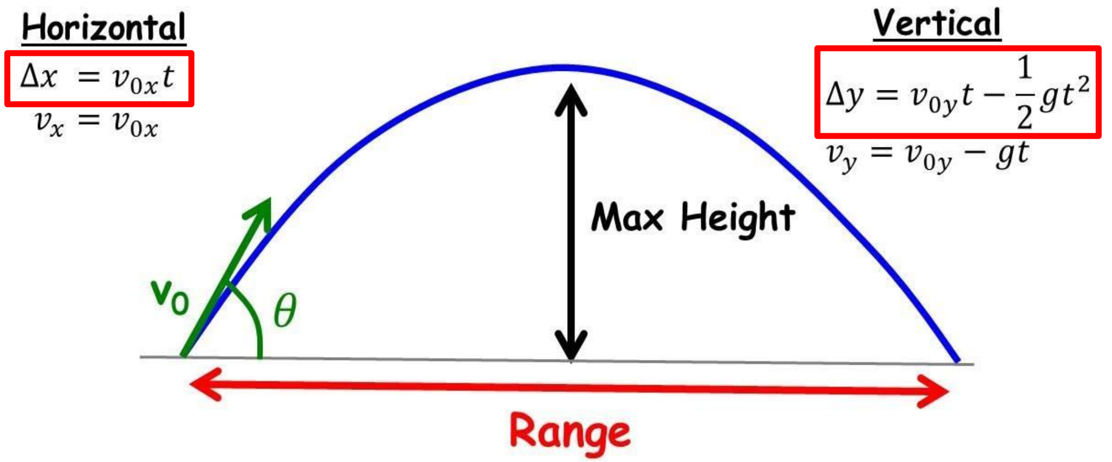




$$\sin\theta = \frac{opp}{hyp} = \frac{v_y}{v} \Rightarrow v_y = v\sin\theta$$

$$\cos\theta = \frac{adj}{hyp} = \frac{v_x}{v} \Rightarrow v_x = v\cos\theta$$

More on projectile motion



$$x = v_0 \cos\theta t \Rightarrow t = \frac{x}{v_0 \cos\theta}$$

$$y = v_0 \sin\theta t - \frac{1}{2} g t^2 = v_0 \sin\theta \left(\frac{x}{v_0 \cos\theta}\right) - \frac{1}{2} g \left(\frac{x}{v_0 \cos\theta}\right)^2$$

$$y = x \tan\theta - \frac{g}{2 v_0^2 \cos^2\theta} x^2$$

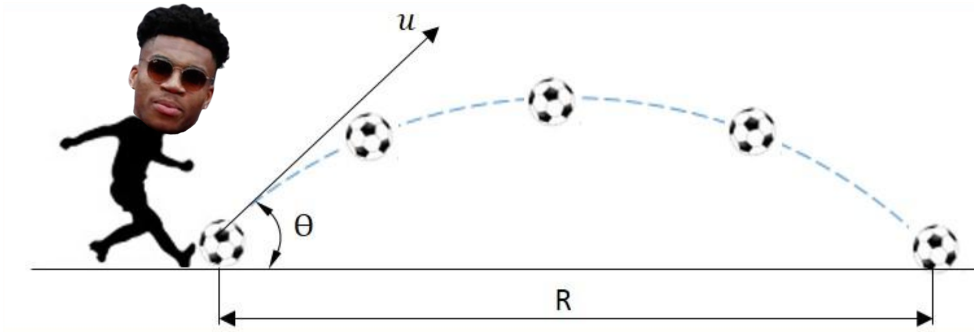
Equation of a parabola!
$y = ax^2 + bx + c$

Note: the boxed equations can be used to show why things move in parabolas when they are thrown

Practice problem #5

Giannis Sina Ugo Antetokounmpo is the best basketball player in the world. He is such a god that he can also play soccer. Giannis is trying to punt the soccer ball as far as possible. He kicks the ball with an initial velocity of 10 m/s at an angle of 45°.

a) How far does the ball go?
b) What is the max height the ball reaches?



1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

a) x-direction:

$$\Delta x = v_0 \cos\theta t$$

- If we can figure out t, then we can get the answer
- t is the time the ball stays in the air

y-direction:

$$v_y = v_{0,y} + at' = v_0 \sin\theta - gt'$$

$$t' = -\frac{v_y - v_0 \sin\theta}{g},\ v_y = 0\ m/s \Rightarrow t' = -\frac{0 - v_0 \sin\theta}{g} = \frac{v_0 \sin\theta}{g}$$

$$t = 2t' = \frac{2v_0 \sin\theta}{g}$$

- The idea is to use the y direction to somehow figure out t
- Trick: find the time it takes to travel half of the trajectory t', then multiply by 2

Alternate method: overall y displacement is Δy = 0 m!

$$\Delta y = 0 = v_0 \sin\theta t - \frac{1}{2} g t^2$$

$$v_0 \sin\theta t = \frac{1}{2} g t^2 \Rightarrow t = \frac{2v_0 \sin\theta}{g}$$

Putting it all together:

$$\Delta x = v_0 \cos\theta\left(\frac{2v_0 \sin\theta}{g}\right) = \frac{2v_0^2 \sin\theta\cos\theta}{g} = \boxed{\frac{v_0^2 \sin(2\theta)}{g}}$$

$$\Delta x = \frac{(10\ m/s)^2 \sin(2*45^\circ)}{9.8\ m/s^2} = \boxed{10.2\ m}$$

This is the range formula!

b) Similar trick to part a: consider half of the trajectory

$$v_y = v_{0,y} + at' = v_0 \sin\theta - gt'$$

$$t' = -\frac{v_y - v_0 \sin\theta}{g},\ v_y = 0\ m/s \Rightarrow t' = -\frac{0 - v_0 \sin\theta}{g} = \frac{v_0 \sin\theta}{g}$$

$$\Delta y = v_0 \sin\theta\left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta}{g}\right)^2$$

$$= \frac{v_0^2 \sin^2\theta}{g} - \frac{v_0^2 \sin^2\theta}{2g} = \boxed{\frac{v_0^2 \sin^2\theta}{2g}}$$

This is the max height formula!

$$\Delta y = \frac{(10\ m/s)^2 \sin^2(45^\circ)}{2*9.8\ m/s^2} = \boxed{2.55\ m}$$

Notice how we only looked at the y direction to solve this part!