2D Kinematics Problems Key

Fun With Fiziks

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Practice Problems Key

1. (a) 0.72 s
   (b) 2.55 m
2. (a) 44.27 m/s
   (b) 2.34 s
   (c) 6.7 m
3. These problems deal with projectile motion!

   (a) We know that the range of a projectile R is given by

   $$R = \frac{v_0^2 \sin 2\theta}{g}$$

   Plugging in the numbers,

   $$R = \frac{2^2 \cdot \sin(2 \cdot 30°)}{9.8} = 0.35\,m$$

   Since $R = 0.35\,m < 10\,m$, $\boxed{\text{the basketball does not reach Andrew.}}$

   Note: To prove the range formula, recall that we can find the total time the ball is in the air t. Then, $R = \Delta x = v_x t = v_0 \cos\theta t$. To find t, notice that the y displacement $\Delta y = 0$. Then, t is

   $$\Delta y = v_y t - \frac{1}{2}gt^2 \Rightarrow 0 = v_0 \sin\theta - \frac{1}{2}gt^2 \Rightarrow t = \frac{2v_0 \sin\theta}{g}$$

   Therefore, we can plug this in to find R.

   $$R = v_0 \cos\theta \Big(\frac{2v_0 \sin\theta}{g}\Big) = \frac{2v_0^2 \sin\theta \cos\theta}{g} = \boxed{\frac{v_0^2 \sin 2\theta}{g}}$$

Knowns	Unknowns
$\Delta x = 4\,m$	$v_0 = ?\,m/s$
$\Delta y = h_{hoop} - h_{Andrew} = 1.5\,m$	
$\theta = 60°$	

x-direction:	y-direction:
$\Delta x = v_x t = v_0 \cos\theta t$	$\Delta y = v_y t - \frac{1}{2}gt^2 = v_0 \sin\theta t - \frac{1}{2}gt^2$
$\therefore x = v_0 \cos\theta t$	$\therefore y = v_0 \sin\theta t - \frac{1}{2}gt^2$

(b) Here is the information we are given in the problem:
Since we don't know what t is, we can eliminate it by solving the x-direction equation for t and substitute it into the y-direction equation.

$$y = v_0 \sin\theta\left(\frac{x}{v_0\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{v_0\cos\theta}\right)^2 = x\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2$$

Note: This is the equation of the parabola that a projectile follows! If you knew this equation, you could have started here and skipped the derivation.

We can plug in all the numbers and solve for v_0.

$$1.5 = 4\tan(60°) - \frac{9.8}{2\cdot v_0^2\cos^2(60°)}\cdot 4^2 \Rightarrow v_0 = \boxed{7.6\,m/s}$$

(c) Here is the information we are given in the problem:

Knowns	Unknowns
$\Delta x = 5\,m$	$t = ?\,s$
$v_0 = 10\,m/s$	
$\theta = 10°$	
$t_{reaction} = 0.15\,s$	

Notice that all of the things we know relate to the x-direction. Then, the x-component of v_0 is $v_x = v_0\cos\theta$. Since we have Δx, v_x, and t, we can use $\Delta x = v_x t = v_0\cos\theta t$. Plugging in numbers and solving for t,

$$t = \frac{\Delta x}{v_0\cos\theta} = \frac{5}{10\cos(10°)} = 0.51\,s$$

Since $t = 0.51\,s > t_{reaction} = 0.15\,s$, $\boxed{\text{Andrew dodges in time.}}$

4. We can split this problem into two parts: before and after the ball bounces off the wall. Before the ball bounces, notice that the ball has to travel a distance of $10\,m$. Also, the initial and final heights are the same, so we can use the range formula.

$$R = 10 = \frac{v_0^2\sin 2\theta}{g} \Rightarrow v_0^2\sin 2\theta = 10g \qquad (1)$$

Right now, we have two unknowns (v_0 and θ), so we will need another equation. We can get another equation when we look at what happens after the ball bounces. We know the equation of the parabola a projectile follows is

$$y = x\tan\theta - \frac{g}{2v_0^2\cos^2\theta}x^2$$

Plugging the numbers and the point $(x, y) = (-2, -5)$ in, we get

$$-5 = -2\tan\theta - \frac{4g}{2v_0^2\cos^2\theta} \Rightarrow 5 = 2\tan\theta + \frac{4g}{2v_0^2\cos^2\theta} \tag{2}$$

Now, we have two equations and two unknowns, so we can solve for v_0 and θ. If this system of equations has no solution, then Justin's statement is correct. Otherwise, Justin is wrong. To solve this system, we can first divide by $\sin\theta$ on both sides of (2).

$$\frac{5}{\sin\theta} = \frac{2}{\cos\theta} + \frac{4g}{2v_0^2\cos^2\theta\sin\theta}$$

By (1), we have $v_0^2\sin 2\theta = 2v_0^2\sin\theta\cos\theta = 10g$. Substituting this in,

$$\frac{5}{\sin\theta} = \frac{2}{\cos\theta} + \frac{4g}{10g\cos\theta} = \frac{2}{\cos\theta} + \frac{2}{5\cos\theta} = \frac{12}{5\cos\theta}$$

At this point, its looking like we are going to get a solution for this system of equations!

$$\frac{5}{\sin\theta} = \frac{12}{5\cos\theta} \Rightarrow \theta = \tan^{-1}\left(\frac{25}{12}\right) = \boxed{64.4^\circ}$$

Plugging $\theta = 64.4^\circ$ into (1) and solving for v_0,

$$v_0^2\sin(2\cdot 64.4^\circ) = 10g \Rightarrow v_0 = \boxed{11.2\ m/s}$$

Therefore, we have concluded that it is possible to make it into the basket, and Justin just needs to get better at the game.